Homework 7

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Due March 16.

1. Consider the wide-sense stochastic process $u = \xi e^{it}$ where ξ is a Gaussian variable with mean 0 and variance 1. What is its stochastic Fourier transform? what is the measure $\rho(dk)$?

2. Consider a stochastic process of the form $u(\omega, t) = \sum_j \xi_j e^{i\lambda_j t}$, where the sum is finite and the ξ_j are independent random variables with means 0 and variances v_j. Calculate the limit as $T \to \infty$ of the random variable $(1/T)\int_{-T}^{T} |u(\omega, s)|^2 ds$. How is it related to the spectrum as we have defined it? What is the limit of $(1/T)\int_{-T}^{T} u ds$?

3. Suppose you have to construct on the computer (for example, for the purpose of modeling the random transport of pollutants) a Gaussian stationary stochastic process with mean 0 and a given covariance function $R(t_2 - t_1)$. Propose a construction (you do not have to implement it).

4. Find a stationary (wide sense) stochastic process $u = u(\omega, t)$ which satisfies (for each ω) the differential equation $y'' + 4y = 0$ with the requirement that $y(t = 0) = 1$.

5. Find the first 3 Hermite polynomials H_0, H_1, H_2 (H_i is a polynomial of degree i, the family is orthonormal with respect to the inner product $(u, v) = \int_{\infty}^{+\infty} e^{-x^2/2} u(x)v(x)dx/\sqrt{2\pi}$).

6. Let η be a random variable. Its characteristic function is defined as $\phi(\lambda) = E[e^{i\lambda\eta}]$. Show that $\phi(0) = 1$, and that $|\phi(\lambda)| \leq 1$ for all λ. Show that if $\phi_1, \phi_2, \ldots, \phi_n$ are the characterstic functions of independent random variables $\eta_1, \ldots, \eta_n$, then the characteristic function of the sum of these variables is the product of the ϕ_i.